$N = 2$ Supergravity Models with Gauge Kac-Moody Groups

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Abstract

In this paper we consider a class of models for vector and hypermultiplets, interacting with $N = 2$ supergravity, with gauge groups being an infinite-dimensional Kac-Moody groups. It is shown that specific properties of Kac-Moody groups, allowing the introduction of the vector fields masses without the usual Higgs mechanism, make it possible to break simultaneously both the supersymmetry and the gauge symmetry. Also, a kind of inverse Higgs mechanism can be realized, that is, in the considered model there exists a possibility to lower masses of the scalar fields, which usually acquire huge masses as a result of supersymmetry breaking. That allows one to use them, for example, as Higgs fields at the second step of the gauge symmetry breaking in the unified models.

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Introduction

One of the most serious problems, which arises when one deals with the exploration of the phenomenological supergravity models, is the problem of the simultaneous breaking of the supersymmetry and the gauge symmetry. Attempts to break the gauge symmetry by means of the usual Higgs mechanism often fail, because both in $N = 1$ supergravity models and in extended supergravity ones all the particles, which could play the role of the Higgs particles, acquire as a rule masses of the order of supersymmetry breaking scale. And if in the case of $N = 1$ supergravity in some models it turns out to be possible to obtain spontaneous gauge symmetry breaking due to radiative corrections, for the extended supergravities, where mass scales of the supersymmetry breaking and, correspondingly, masses of the Higgs particles are essentially larger, it hardly works.

Let us reconsider the possibilities to have spontaneous gauge symmetry breaking. As is well known, the key element of all models is the gauge invariant description of massive vector particles, which is possible due to the introduction of the Goldstone scalar field with inhomogeneous transformation law. For the Abelian vector field the Lagrangian has a very simple form

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{m^2}{2}A_\mu^2 - mA_\mu \partial_\mu \phi + \frac{1}{2}(\partial_\mu \phi)^2 \tag{1}$$

being invariant under the following gauge transformations: $\delta A_\mu = \partial_\mu \varepsilon$ and $\delta \phi = m\varepsilon$.

If one starts from the analogous Lagrangian and gauge transformations in the simplest case of the non-abelian $SU(2)$ gauge group:

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{m^2}{2}(A_\mu^a)^2 - mA_\mu^a \partial_\mu \phi^a + \frac{1}{2}(\partial_\mu \phi^a)^2 \\
F_{\mu\nu}^a &= \partial_\mu A_\nu - g\varepsilon^{abc} A_\mu^b A_\nu^c - (\mu \leftrightarrow \nu) \\
\delta_0 A_\mu^a &= (\partial_\mu \delta^{ab} - g\varepsilon^{abc} A_\mu^b)\varepsilon^c \qquad \delta_0 \phi^a = m\varepsilon^a
\end{aligned} \tag{2}$$

and try to complete both the interaction Lagrangian and the transformation law of the field ϕ, requiring the full Lagrangian to be gauge invariant, one will see that there exist two possible scenarios [1]. If we proceed without introducing any other scalar fields we necessarily will come to the gauge invariant description of massive vector fields where scalars realize a non-linear σ-model [2].

There is another possibility, leading to the ordinary model of the spontaneous breaking of $SU(2)$ gauge group through the Higgs mechanism. To obtain the corresponding formulas, one has to introduce additional scalar field χ with the transformation law $\delta\chi = \frac{g}{2}\phi^a \varepsilon^a$, which together with the fields ϕ^a forms complex $SU(2)$-dublet, thus avoiding a non-linear realization.

Therefore, apart from the usual Higgs mechanism, one can exploit the fact, that in the supergravity theories the scalar fields often realize non-linear σ-models of the form G/H and the gauging of the isometries in such models necessarily leads to the gauge symmetry breaking. Indeed there are examples of the supergravity models of such a kind (see e.g. [3, 4] for $N = 2$ case), but in many $N = 1$ and in all extended supergravities one deals with the non-compact groups G, moreover the choice of possible gauge groups is highly restricted.

So, the generalization to the non-abelian case leads either to the non-linear models, or to the Higgs mechanism and both of these schemes fail in the extended supergravity

models. But really there exists a third possibility connected with the infinite-dimensional groups of the Kac-Moody type. Such groups arise in a natural way when one deals with the compactifications from higher dimensions and also in attempts to obtain an effective field theory for superstrings (e.g. [5, 6]). But in this paper we will not rely on any geometric interpretation and will just investigate $N = 2$ supergravity models with the gauge Kac-Moody groups in the same spirit as in the [7, 8, 9]. In the next Section we first of all reproduce the rather well known formulas for the gauge theory based on the usual affine Kac-Moody groups and consider the introduction of the mass terms for the appropriate vector fields. All the formulas, of course, are similar to those we will get if we consider the five-dimensional Yung-Mills theory and then compactify the fifth dimension on the circle. But we stress that in sharp contrast with the finite dimensional gauge groups the introduction of the mass terms appears to be as simple matter as in the abelian case — there is no need in the Higgs fields with any non-trivial potential. This allows us to construct a generalization of the simplest models we started with which could mimic the spontaneous gauge symmetry breaking $G \to H$, where for example one can have $G = SU(5)$ and $H = SU(3) \otimes SU(2) \otimes U(1)$.

In Section 2, as a preliminary step to the local $N = 2$ supersymmetry, we consider the case of the global one. In this, we choose to work with massive vector multiplets without central charges. The reason is that in the $N = 2$ supergravity the central charges are necessarily gauged (see, e.g., [10]), the gauge fields being graviphotons. But the graviphotons play a very essential role in the spontaneous supersymmetry breaking, so it would be hard to have simultaneous breaking of the gauge symmetry and supersymmetry.

In Section 3 we consider an interaction of our globally $N = 2$ supersymmetric models with gauge Kac-Moody groups and the $N = 2$ supergravity and investigate the possibilities of spontaneous symmetry breaking in such models. The main results of our investigations are twofold. First, we show that it is indeed possible to have simultaneous breaking of gauge as well as supersymmetries and calculate the mass spectrum that appears after such a breaking have taken place. Second, we will see that a kind of inverse Higgs effect arises — not only the fields which were massless could gain masses as a result of supersymmetry breaking, but some of the initially massive fields could become light or even massless. It is interesting to note that for such a mechanism to be operative the scale of the gauge symmetry breaking and the one for the supersymmetry breaking have to be close to each other.

1 Kac-Moody groups and gauge symmetry breaking

The affine Kac-Moody algebra without the central charge has the following commutation relations:

$$[T^a_m, T^b_n] = f^{abc} T^c_{m+n}, \tag{3}$$

where $n, m \in \mathbf{Z}$, $T^a_0 \in G$ for any semisimple Lie algebra G with structural constants f^{abc}, so $1 < a, b, c < \mathrm{dim}G$. Let us assume the generators of this algebra to be antihermitian:

$$(T^a_m)^+ = -T^a_{-m}. \tag{4}$$

Let us consider a gauge field that lies in the algebra (3):

$$\mathcal{A}_\mu = A_{\mu m}{}^a T^a_{-m} \qquad \mathcal{A}^+_\mu = -\mathcal{A}_\mu \qquad (\mathcal{A}_{\mu m}{}^a)^* = A_{\mu -m}{}^a \tag{5}$$

The associated field strength has the usual form:

$$\mathcal{F}_{\mu\nu} = [\nabla_\mu, \nabla_\nu], \tag{6}$$

where $\nabla_\mu = \partial_\mu + \mathcal{A}_\mu$.

Under infinitesimal gauge transformations with parameter ε also lying in algebra (3) the gauge field $\mathcal{A}_\mu$ and the field strength $\mathcal{F}_{\mu\nu}$ transform as the following:

$$\begin{aligned} \delta\mathcal{A}_\mu &= [\nabla_\mu, \varepsilon] = \partial_\mu\varepsilon + [\mathcal{A}_\mu, \varepsilon] \\ \delta\mathcal{F}_{\mu\nu} &= [\mathcal{F}_{\mu\nu}, \varepsilon]. \end{aligned} \tag{7}$$

The Lagrangian, invariant under these transformations, has the form, that coincides with the case of the finite dimensional gauge group:

$$\mathcal{L} = \frac{1}{8}\mathrm{Sp}\{\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}\}, \tag{8}$$

where $\mathrm{Sp}\{T^a_m T^b_n\} = -2\delta^{ab}\delta(m+n)$ with the notation $\delta(m) = \left\{\begin{smallmatrix} 0 & \text{at} & m\neq 0 \\ 1 & \text{at} & m=0 \end{smallmatrix}\right.$.

Now one can rewrite all the formulas, obtained above, in the components:

$$\begin{aligned} \mathcal{F}_{\mu\nu} &= F_{\mu\nu}{}^a_m T^a_{-m} \qquad\qquad \varepsilon = \varepsilon^a_m T^a_{-m} \\ F_{\mu\nu}{}^a_m &= \partial_\mu A_\nu{}^a_m - \partial_\nu A_\mu{}^a_m + f^{abc} A_\mu{}^b_n A_\nu{}^c_{m-n} \end{aligned} \tag{9}$$

$$\begin{aligned} \delta A_\mu{}^a_m &= \partial_\mu\varepsilon^a_m + f^{abc} A_\mu{}^b_n \varepsilon^c_{m-n} \\ \delta F_{\mu\nu}{}^a_m &= f^{abc} F_{\mu\nu}{}^b_n \varepsilon^c_{m-n} \end{aligned} \tag{10}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}{}^a_m F_{\mu\nu}{}^a_{-m}. \tag{11}$$

In order to consider spontaneous symmetry breaking, let us introduce scalar field ϕ, lying in algebra (3):

$$\phi = \phi^a_m T^a_{-m} \qquad \phi^+ = -\phi \qquad (\phi^a_m)^* = \phi^a_{-m} \tag{12}$$

Under the infinitesimal gauge transformations this field transforms according to the usual rule:

$$\delta\phi = [\phi, \varepsilon] \tag{13}$$

and covariant derivative has the form:

$$D_\mu\phi = [\nabla_\mu, \phi] = \partial_\mu\phi + [\mathcal{A}, \phi]. \tag{14}$$

In the components all these formulas take the following form:

$$\delta\phi^a_m = f^{abc}\phi^b_n\varepsilon^c_{m-n} \tag{15}$$

$$D_\mu\phi^a_m = \partial_\mu\phi^a_m + f^{abc} A_\mu{}^b_n \phi^c_{m-n}. \tag{16}$$

The total Lagrangian, invariant under the gauge transformations (10, 15), is the following:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}{}^a_m F_{\mu\nu}{}^a_{-m} + \frac{1}{2} D_\mu\phi^a_m D_\mu\phi^a_{-m}. \tag{17}$$

Now let us modify the gauge transformation of the field ϕ. Namely, let us introduce inhomogeneous term in transformations (15):

$$\delta\phi^a_m = f^{abc}\phi^b_n\varepsilon^c_{m-n} + i\mu m\varepsilon^a_m \tag{18}$$

Covariant derivative also changes its form:

$$D_\mu\phi^a_m = \partial_\mu\phi^a_m + f^{abc}A_{\mu}{}^b_n\phi^c_{m-n} - i\mu m A_{\mu}{}^a_m \tag{19}$$

In this, Lagrangian (17) with covariant derivative $D_\mu\phi$, defined in (19), is invariant under the gauge transformations (10, 18). Let us stress, that the fact we are working with the infinite-dimensional algebra is crucial for the possibility to have such a gauge invariance with inhomogeneous terms. As we have already mentioned for any finite-dimensional algebra the introduction of these inhomogeneous terms either leads to the non-linear σ-models or requires the presence of the Higgs fields.

It can be easily seen, that the vector fields $A_{\mu}{}^a_m$ with $m \neq 0$ acquire masses, due to the following mass term (arising as usual from the covariant derivatives in the scalar field kinetic terms):

$$\mathcal{L}_M = \frac{\mu^2 m^2}{2} A_{\mu}{}^a_m A_{\mu}{}^a_{-m} = \frac{\mu^2 m^2}{2} A_{\mu}{}^a_m (A_{\mu}{}^a_m)^*. \tag{20}$$

So, we have spontaneous breaking of the total Kac-Moody group to its subgroup G, in this, the vector fields acquire masses, proportional to the level number m and to the symmetry breaking scale μ.

But we are interested in the fields from the lowest level, which we associate with the observable particles. At this level gauge group G remains unbroken and corresponding vector fields remain massless. In order to have spontaneous symmetry breaking, under which some of the vector fields from the lowest level acquire masses, we should generalize algebra (3).

Let us assume, that group G has some subgroup H with generators T^a, $a = 1, ..., \mathrm{dim}H$, all the other generators of the group G we denote as $T^{a'}$, $a' = \mathrm{dim}H + 1, ..., \mathrm{dim}G$. Let the commutation relations of this algebra are such that it admits a $\mathbf{Z}_2$-grading, i.e.:

$$[T^a, T^b] = f^{abc}T^c \qquad [T^{a'}, T^{b'}] = f^{a'b'c}T^c \qquad [T^a, T^{b'}] = f^{ab'c'}T^{c'} \tag{21}$$

For any such algebra it is not difficult to construct an infinite dimensional algebra which will be the generalization of simplest case described above. Namely, all the Jacoby identities will hold if one assigns the integer levels to the generators of subgroup H — T^a_m and half-integer ones to other generators $T^{a'}_{m+1/2}$. Corresponding commutation relations have the following form:

$$[T^a_m, T^b_n] = f^{abc}T^c_{m+n} \quad [T^a_n, T^{b'}_{m+1/2}] = f^{ab'c'}T^{c'}_{m+n+1/2} \quad [T^{a'}_{m+1/2}, T^{b'}_{n+1/2}] = f^{a'b'c}T^c_{m+n+1} \tag{22}$$

For the gauge field

$$\mathcal{A}_\mu = A_{\mu}{}^a_m T^a_{-m} + A_{\mu}{}^{a'}_{m+1/2} T^{a'}_{-(m+1/2)} \tag{23}$$

lying in algebra (22), expressions for field strength and the gauge transformations in the components are the following:

$$\begin{aligned} F_{\mu\nu}{}^a_m &= \partial_\mu A_{\nu}{}^a_m - \partial_\nu A_{\mu}{}^a_m + f^{abc}A_{\mu}{}^b_n A_{\nu}{}^c_{m-n} + f^{ab'c'}A_{\mu}{}^{b'}_{n+1/2}A_{\nu}{}^{c'}_{m-(n+1/2)} \\ F_{\mu\nu}{}^{a'}_{m+1/2} &= \partial_\mu A_{\nu}{}^{a'}_{m+1/2} - \partial_\nu A_{\mu}{}^{a'}_{m+1/2} + f^{a'b'c}(A_{\mu}{}^{b'}_{m+1/2-n}A_{\nu}{}^c_n - [\mu \leftrightarrow \nu]) \end{aligned} \tag{24}$$

$$\begin{aligned}\delta A_{\mu m}^{\ a} &= \partial_\mu \varepsilon_m^a + f^{abc} A_{\mu n}^{\ b} \varepsilon_{m-n}^c + f^{ab'c'} A_{\mu n+1/2}^{\ b'} \varepsilon_{m-(n+1/2)}^{c'} \\ \delta A_{\mu m+1/2}^{\ a'} &= \partial_\mu \varepsilon_{m+1/2}^{a'} + f^{a'b'c} (A_{\mu n+1/2}^{\ b'} \varepsilon_{m-n}^c - A_{\mu n}^{\ c} \varepsilon_{m+1/2-n}^{b'}). \end{aligned} \tag{25}$$

Expressions for the covariant derivative and the gauge transformations of the scalar field ϕ, lying in algebra (22), in the components have the following form:

$$\begin{aligned} D_\mu \phi_m^a &= \partial_\mu \phi_m^a + f^{abc} A_{\mu n}^{\ b} \phi_{m-n}^c + f^{ab'c'} A_{\mu n+1/2}^{\ b'} \phi_{m-(n+1/2)}^{c'} + i\mu m \varepsilon_m^a \qquad (26) \\ D_\mu \phi_{m+1/2}^{a'} &= \partial_\mu \phi_{m+1/2}^{a'} + f^{a'b'c} (A_{\mu n+1/2}^{\ b'} \phi_{m-n}^c - A_{\mu n}^{\ c} \phi_{m+1/2-n}^{b'}) + i\mu(m+1/2) \varepsilon_{m+1/2}^{a'} \end{aligned}$$

$$\begin{aligned} \delta \phi_m^a &= f^{abc} \phi_n^b \varepsilon_{m-n}^c + f^{ab'c'} \phi_{n+1/2}^{b'} \varepsilon_{m-(n+1/2)}^{c'} \\ \delta \phi_{m+1/2}^{a'} &= f^{a'b'c} (\phi_{n+1/2}^{b'} \varepsilon_{m-n}^c - \phi_n^c \varepsilon_{m+1/2-n}^{b'}. \end{aligned} \tag{27}$$

The total Lagrangian, invariant under the gauge transformations (25, 27), is the following:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu}{}_m^a F_{\mu\nu}{}_{-m}^a - \frac{1}{4} F_{\mu\nu}{}_{m+1/2}^{a'} F_{\mu\nu}{}_{-(m+1/2)}^{a'} + \\ &\quad + \frac{1}{2} D_\mu \phi_m^a D_\mu \phi_{-m}^a + \frac{1}{2} D_\mu \phi_{m+1/2}^{a'} D_\mu \phi_{-(m+1/2)}^{a'}. \end{aligned} \tag{28}$$

The mass terms for the vector fields take the form:

$$\mathcal{L}_M = \frac{\mu^2 m^2}{2} A_{\mu m}^{\ a} A_{\mu -m}^{\ a} + \frac{\mu^2}{2} (m+1/2)^2 A_{\mu m+1/2}^{\ a'} A_{\mu m+1/2}^{\ a'}. \tag{29}$$

It is seen that from the fields of the lowest level the fields $A_{\mu 0}^{\ a}$, lying in the subgroup H, remain massless, while the fields $A_{\mu 1/2}^{\ a'}$ acquire masses $\mu/2$. Hence, such a theory could indeed mimic the spontaneous gauge symmetry breaking $G \to H$, for example, $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$. In this, one would still have usual relations for three gauge coupling constants.

2 $N = 2$ supersymmetry model

As a preliminary step to a $N = 2$ supergravity model let us consider $N = 2$ supersymmetry model with the mechanism of the gauge symmetry breaking, described in the previous section. Here we are not interested in the problem of the supersymmetry breaking and are investigating, in which way vector fields acquire masses in a supersymmetric model with a gauge Kac-Moody group.

There are two ways to describe a massive vector $N = 2$ supermultiplet [11]. In the first case the scalar Goldstone boson belongs to another vector multiplet. This case leads to the so called massive vector multiplets with central charge. As we have already mentioned, in the $N = 2$ supergravity the central charge will necessarily be gauged, the gauge field being graviphoton. As the graviphoton plays a very special role in our mechanism of spontaneous supersymmetry breaking, we will not consider such multiplets in this paper.

In the second case Goldstone boson belongs to a hypermultiplet and the central charge does not arise. To describe the corresponding model let us consider some number of the vector

multiplets $(A_\mu^M, \Theta_i^M, \mathcal{Z}^M = \mathcal{X}^M + \gamma_5 \mathcal{Y}^M)$ and hypermultiplets $(\Omega^{iM}, X^M, L^{aM})$, carrying the same index M, where $i = 1, 2$ and $a = 1, 2, 3$. The scalar fields X^M and $\vec{L}^M$ of the hypermultiplet transform as a singlet and a triplet under the $SU(2)$ automorphism group of the superalgebra. Such a description of the hypermultiplets would enable us to consider the fields X^M as Goldstone ones without breaking the $SU(2)$ invariance.

The $N = 2$ supersymmetric Lagrangian, before switching on the gauge interactions, have the following form:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}(A_\mu)^2 + \frac{i}{2}\bar{\Theta}_i \hat{\partial} \Theta_i + \frac{1}{2}\partial_\mu \bar{\mathcal{Z}} \partial_\mu \mathcal{Z} + \\ &\quad + \frac{i}{2}\bar{\Omega}^i \hat{\partial} \Omega^i + \frac{1}{2}(\partial_\mu X)^2 + \frac{1}{2}(\partial_\mu \vec{L})^2. \end{aligned} \tag{30}$$

Supertransformations of the fields from both multiplets, under which Lagrangian (30) is invariant, are the following:

$$\begin{aligned} &\delta A_\mu = i(\bar{\Theta}_i \gamma_\mu \eta_i) \qquad \delta \Theta_i = -\frac{1}{2}(\sigma A)\eta_i - i\varepsilon_{ij} \hat{\partial} \mathcal{Z} \eta_j \\ &\delta \mathcal{X} = \varepsilon^{ij}(\bar{\Theta}_i \eta_j) \qquad \delta \mathcal{Y} = \varepsilon^{ij}(\bar{\Theta}_i \gamma_5 \eta_j) \end{aligned} \tag{31}$$

$$\begin{aligned} &\delta \Omega^i = -i(\hat{\partial} X \delta_i{}^j + \hat{\partial} L_i{}^j)\eta_i \\ &\delta X = (\bar{\Omega}^i \eta_i) \qquad \delta \vec{L} = (\bar{\Omega}^i \vec{\tau}_i{}^j \eta_j), \end{aligned} \tag{32}$$

where $\vec{\tau}_i{}^j$ are Pauli matrices and the following notation was introduced: $L_i{}^j = \vec{L}\vec{\tau}_i{}^j$.

In order to switch on the gauge interaction in this model, let us assume that all the fields from the vector and the hypermultiplets transform under adjoint representation of some group G with the structural constants f^{MNK}. The following substitutions in Lagrangian (30) make this Lagrangian gauge invariant:

$$\partial_\mu \mathcal{Z}^M \to \partial_\mu \mathcal{Z}^M + f^{MNK} A_\mu^N \mathcal{Z}^K \tag{33}$$

with the analogous expressions for the other fields derivatives. In order to restore the supersymmetry invariance, one has to add the following terms to the Lagrangian and the supertransformation laws:

$$\begin{aligned} \mathcal{L}' &= f^{MNK}\left\{-\frac{1}{2}\varepsilon^{ij}(\bar{\Theta}_i^M \mathcal{Z}^N \Theta_j^K) + (\Theta_i^M X^N \Omega^{jK}) + (\bar{\Theta}_i^M L_j^{Ni} \Omega^{jK}) + \frac{1}{2}\varepsilon_{ij}(\bar{\Omega}^{iM} \mathcal{Z}^N \Omega^{jK})\right\} + \\ &\quad + \frac{1}{8}(f^{MNK} \bar{\mathcal{Z}}^N \mathcal{Z}^K)^2 - \frac{1}{2}|f^{MNK} X^N \mathcal{Z}^K|^2 - \frac{1}{2}|f^{MNK} \vec{L}^N \mathcal{Z}^K|^2 - \frac{1}{2}(\Delta^{aM})^2 \end{aligned} \tag{34}$$

$$\begin{aligned} \delta' \Theta_i^M &= \frac{1}{2} f^{MNK} \bar{\mathcal{Z}}^N \mathcal{Z}^K \eta_i - \Delta_i^{Mj} \eta_j \\ \delta' \Omega^{iM} &= \varepsilon^{ij} f^{MNK} \mathcal{Z}^N (X^K \delta_j{}^k + L_j^{Kk}) \eta_k, \end{aligned} \tag{35}$$

where the following notation is used:

$$\Delta^{aM} = f^{MNK}(X^N L^{aK} - \frac{1}{2}\varepsilon^{abc} L^{bN} L^{cK}) \tag{36}$$

To demonstrate in this model the mechanism of the gauge symmetry breaking, described in the previous section, let us assume that all the fields lie in the Kac-Moody algebra (3) rather than a finite Lie algebra and divide index M into a pair of indices $\{A, m\}$, where A is an index of adjoint representation of the finite group G and m is an infinite index of the Kac-Moody algebra. In this, structural constants take the form:

$$f^{MNK} = f^{ABC}\delta(m+n+k) \tag{37}$$

and the summing rule has, for example, the following form: $\partial_\mu \bar{\mathcal{Z}}^M \partial_\mu \mathcal{Z}^M = \partial_\mu \bar{\mathcal{Z}}^A_m \partial_\mu \mathcal{Z}^A_{-m}$. Under the gauge transformations all the fields except the fields X^A_m transform according to formulas (10, 15) of the previous section and transformation laws of the fields X^A_m have an inhomogeneous term (the same as in (18)):

$$\delta X^A_m = f^{ABC} X^B_n \varepsilon^C_{m-n} + i\mu m \varepsilon^A_m. \tag{38}$$

In this, the covariant derivatives of the fields X^A_m are the following:

$$D_\mu X^A_m = \partial_\mu X^A_m + f^{ABC} A_{\mu n}{}^B X^C_{m-n} - i\mu m A_{\mu m}{}^A. \tag{39}$$

In order to restore supersymmetry invariance, broken by the inhomogeneous term in (39), one has to add to the Lagrangian and the supertransformation laws the following terms:

$$\begin{aligned} \mathcal{L}'' &= i\mu m(\bar{\Theta}_{im}{}^A \Omega^{iA}{}_{-m}) - \frac{\mu^2 m^2}{2}\bar{\mathcal{Z}}^A_m \mathcal{Z}^A_{-m} - \frac{\mu^2 m^2}{2}\vec{L}^A_m \vec{L}^A_{-m} - \\ &\quad -\frac{i}{2}(m-n) f^{ABC}(\mathcal{Z}^A_M \bar{\mathcal{Z}}^B_n + \vec{L}^A_m \vec{L}^B_n) x^C_{-m-n}. \end{aligned} \tag{40}$$

In the full correspondence with the non-supersymmetric model of the previous section we have in the model under consideration a spontaneous breaking of the gauge symmetry. Due to the supersymmetry of the model, all the fields, both bosonic and fermionic ones, acquire equal masses. The only exception is the fields X^A_m, which turn out to be Goldstone ones. The mass terms of the model look like:

$$\mathcal{L}_M = \frac{1}{2}\mu^2 m^2 A_{\mu m}{}^A A_{\mu -m}{}^A + i\mu m(\bar{\Theta}_{im}{}^A \Omega^{iA}{}_{-m}) - \frac{\mu^2 m^2}{2}\bar{\mathcal{Z}}^A_m \mathcal{Z}^A_{-m} - \frac{\mu^2 m^2}{2}\vec{L}^A_m \vec{L}^A_{-m}. \tag{41}$$

The invariance of the model under the supertransformations is intact and all the fields can be grouped into the massive $N = 2$ supermultiplets.

Now, it is an easy task to generalize the model considered to the case of the generalized Kac-Moody algebra (22). In this, a part of the vector fields of the lowest level acquire masses and the gauge group G are broken to its subgroup H. Both scalar and spinor fields acquire the same masses as the vector fields because the supersymmetry is unbroken.

3 $N = 2$ supergravity model

In this section we investigate the supergravity generalization of the supersymmetric model described in the previous section. We choose to work with a model of the $N = 2$ supergravity interacting with vector multiplets with the scalar field geometry $SO(2,m)/SO(2) \otimes SO(m)$

and with hypermultiplets with the scalar fields geometry $SO(4,m)/SO(4) \otimes SO(m)$. As it has been shown in [12], such a combination of scalar field geometries admits a spontaneous supersymmetry breaking with two arbitrary scales and without a cosmological term. (Note, that such geometries appear in a natural way in the investigations of $N=2$ $D=4$ superstrings). Moreover, the σ-model chosen for the hypermultiplets is an essential part of the models constructed in [13, 14] where the scalar fields parameterize non-symmetric quaternionic manifolds and, therefore, it allows interesting generalizations.

3.1 Vector multiplets

To describe the interaction of vector multiplets with N=2 supergravity, let us introduce the following fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu i}$, $i=1,2$, Majorana spinors ρ_i, scalar fields $\hat{\varphi}$, $\hat{\pi}$, and $(m+2)$ vector multiplets $\{A_\mu^M, \Theta_i^M, \mathcal{Z}^M = \mathcal{X}^M + \gamma_5 \mathcal{Y}^M\}$, $M=1,2,...m+2$, $g^{MN} = (--,+...+)$. It is not difficult to see that the set of spinor and scalar fields is superfluous (which is necessary for symmetrical description of graviphotons and matter vector fields). The following set of constraints corresponds to the model with the geometry $SO(2,m)/SO(2) \otimes SO(m)$:

$$\bar{\mathcal{Z}} \cdot \mathcal{Z} = -2 \qquad \mathcal{Z} \cdot \mathcal{Z} = 0 \qquad \mathcal{Z} \cdot \Theta_i = \bar{\mathcal{Z}} \cdot \Theta_i = 0. \tag{42}$$

The number of physical degrees of freedom is correct only when the theory is invariant under the local $O(2) \approx U(1)$ transformations, the combination $(\bar{\mathcal{Z}}\partial_\mu \mathcal{Z})$ playing the role of a gauge field. Covariant derivatives for scalar fields $\mathcal{Z}$ and $\bar{\mathcal{Z}}$ look like

$$D_\mu = \partial_\mu \pm \frac{1}{2}(\bar{\mathcal{Z}}\partial_\mu \mathcal{Z}), \tag{43}$$

where covariant derivative $D_\mu \mathcal{Z}$ has the sign "+" and $D_\mu \bar{\mathcal{Z}}$ has the sign "-".

In the given notations the Lagrangian of the interaction looks as follows:

$$\begin{aligned}
\mathcal{L}^F &= \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_{\mu i}\gamma_5\gamma_\nu D_\rho \Psi_{\sigma i} + \frac{i}{2}\bar{\rho}^i \hat{D}\rho_i + \frac{i}{2}\bar{\Theta}^i \hat{D}\Theta_i - \\
&+ e^{\hat{\varphi}/\sqrt{2}} \left\{ \frac{1}{4}\varepsilon^{ij}\bar{\Psi}_{\mu i}(\mathcal{Z}(A^{\mu\nu} - \gamma_5 \tilde{A}^{\mu\nu}))\Psi_{\nu j} + \frac{1}{4}\bar{\Theta}^i\gamma^\mu(\sigma A)\Psi_{\mu i} + \right. \\
&+ \left. \frac{i}{4\sqrt{2}}\bar{\rho}^i\gamma^\mu(\mathcal{Z}(\sigma A))\Psi_{\mu i} + \frac{\varepsilon^{ij}}{8}\left[2\sqrt{2}\bar{\rho}_i(\sigma A)\Theta_j + \bar{\Theta}_i{}^M(\mathcal{Z}(\sigma A))\Theta_j{}^M\right] \right\} \\
&- \frac{1}{2}\varepsilon^{ij}\bar{\Theta}_i{}^M\gamma^\mu\gamma^\nu D_\nu \mathcal{Z}^M \Psi_{\mu j} - \frac{1}{2}\varepsilon^{ij}\bar{\rho}_i\gamma^\mu\gamma^\nu(\partial_\nu\hat{\varphi} + \gamma_5 e^{-\sqrt{2}\hat{\varphi}}\partial_\nu\hat{\pi})\Psi_{\mu j}
\end{aligned} \tag{44}$$

$$\begin{aligned}
\mathcal{L}^B &= -\frac{1}{2}R - \frac{1}{4}e^{\sqrt{2}\hat{\varphi}}\left[A_{\mu\nu}{}^2 + 2(\mathcal{Z}\cdot A_{\mu\nu})(\bar{\mathcal{Z}}\cdot A_{\mu\nu})\right] - \frac{\hat{\pi}}{2\sqrt{2}}(A\cdot\tilde{A}) + \\
&+ \frac{1}{2}(\partial_\mu\hat{\varphi})^2 + \frac{1}{2}e^{-2\sqrt{2}\hat{\varphi}}(\partial_\mu\hat{\pi})^2 + \frac{1}{2}D_\mu\mathcal{Z}^A D_\mu\bar{\mathcal{Z}}^A.
\end{aligned} \tag{45}$$

Covariant derivatives of the spinor fields have the following form:

$$D_\mu \eta_i = D_\mu^G \eta_i - \frac{1}{4}(\bar{\mathcal{Z}}\partial_\mu\mathcal{Z})\eta_i + \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\hat{\varphi}}\gamma_5\partial_\mu\hat{\pi}\eta_i$$

$$D_\mu\rho_i = D^G_\mu\rho_i + \frac{1}{4}(\bar{\mathcal{Z}}\partial_\mu\mathcal{Z})\rho_i + \frac{3}{2\sqrt{2}}e^{-\sqrt{2}\hat{\varphi}}\gamma_5\partial_\mu\hat{\pi}\rho_i \tag{46}$$
$$D_\mu\Theta_i = D^G_\mu\Theta_i - \frac{1}{4}(\bar{\mathcal{Z}}\partial_\mu\mathcal{Z})\Theta_i - \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\hat{\varphi}}\gamma_5\partial_\mu\hat{\pi}\Theta_i$$

and derivative of the field $\Psi_{\mu i}$ is the same as for η_i.

Supertransformation laws look like:

$$\begin{aligned}
\delta\Theta^M_i &= -\frac{1}{2}e^{\hat{\varphi}/\sqrt{2}}\sigma^{\mu\nu}\left\{A^M + \frac{1}{2}\bar{\mathcal{Z}}^M(\mathcal{Z}A) + \frac{1}{2}\mathcal{Z}^M(\bar{\mathcal{Z}}A)\right\}_{\mu\nu}\eta_i - i\varepsilon_{ij}\hat{D}\mathcal{Z}^M\eta_i \\
\delta\rho_i &= -\frac{1}{2\sqrt{2}}e^{\hat{\varphi}/\sqrt{2}}\mathcal{Z}(\sigma A)\eta_i - i\varepsilon_{ij}\gamma^\mu(\partial_\mu\hat{\varphi} + \gamma_5 e^{-\sqrt{2}\hat{\varphi}}\partial_\mu\hat{\pi})\eta_i \\
\delta\Psi\mu i &= 2D_\mu\eta_i + \frac{i}{4}\varepsilon_{ij}e^{\hat{\varphi}/\sqrt{2}}\bar{\mathcal{Z}}(\sigma A)\eta_i \qquad \delta\hat{\pi} = e^{\sqrt{2}\hat{\varphi}}\varepsilon^{ij}(\bar{\rho}_i\gamma_5\eta_j) \\
\delta\boldsymbol{\mathcal{X}}^A &= \varepsilon^{ij}(\bar{\Theta}_i{}^A\eta_j) \qquad \delta\boldsymbol{\mathcal{Y}}^A = \varepsilon^{ij}(\bar{\Theta}_i{}^A\gamma_5\eta_j) \qquad \delta\hat{\varphi} = \varepsilon^{ij}(\bar{\rho}_i\eta_j) \\
\delta A^A_\mu &= e^{-\hat{\varphi}/\sqrt{2}}\left\{\varepsilon^{ij}(\bar{\Psi}_{\mu i}\mathcal{Z}^A\eta_j) + i(\bar{\Theta}^A_i\gamma_\mu\eta^i) - \frac{i}{\sqrt{2}}(\bar{\rho}^i\gamma_\mu\mathcal{Z}^A\eta_i)\right\}.
\end{aligned} \tag{47}$$

3.2 Hypermultiplets

Now, in order to generalize the supersymmetric model of the previous section, we need a parameterization of the $SO(4,m)/SO(m)\otimes SO(4)$ non-linear σ-model where four scalar fields of the hypermultiplet are divided into the singlet X and the triplet $\vec{L}$. Such a model has been constructed by the authors in [13]. It contains, apart from the fields of $N=2$ supergravity, the following fields: scalar field φ, Majorana spinor fields χ^i and $(m+6)$ hypermultiplets $(X^A, \vec{L}^A, \Omega^{iA})$, $g^{AB} = (-,-,-,+,...+)$, with the following constraints on the fields $\vec{L}$ and Ω^i, corresponding to the scalar field geometry $SO(3,m+3)/SO(3)\otimes SO(m+3)$:

$$L^{aA}L^b_A = -\delta^{ab} \qquad L^{aA}\Omega^i_A = 0 \tag{48}$$

This model is invariant under the local $SO(3)$-transformations with the combination $A^a_\mu = \varepsilon^{abc}(L^{bA}\overleftrightarrow{\partial}_\mu L^{cA})$, playing the role of the gauge field. The corresponding covariant derivatives for the fields $\vec{L}^A$, for example, have the following form:

$$D_\mu L^{aA} = \partial_\mu L^{aA} + L^{bA}(L^{bB}\partial_\mu L^{aB}) \qquad L^{aA}D_\mu L^b_A = 0 \tag{49}$$

As it has been shown in [13], the scalar fields $(\varphi, X^A, \vec{L}^A)$ parameterize quaternionic manifold with geometry $SO(4,m+4)/SO(4)\otimes SO(m+4)$.

The Lagrangian of the model without the terms, describing the pure $N=2$ supergravity, has the form:

$$\begin{aligned}
\mathcal{L}_B &= \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}e^{2\varphi}((\partial_\mu X)^2 + 2(\vec{L}\partial_\mu X)^2) + \frac{1}{2}D_\mu\vec{L}D_\mu\vec{L} + \\
&+\frac{i}{2}\bar{\chi}^i\hat{D}\chi^i + \frac{i}{2}\bar{\Omega}^i\hat{D}\Omega^i - \frac{1}{2}\bar{\Omega}^i\gamma^\mu\gamma^\nu[e^\varphi(\partial_\nu X + \vec{L}(\vec{L}\partial_\nu X))\delta_i{}^j + D_\nu L_i{}^j]\Psi_{\mu j} \\
&-\frac{1}{2}\bar{\chi}^i\gamma^\mu\gamma^\nu(\partial_\nu\varphi\delta_i{}^j - e^\varphi(L_i{}^j\partial_\nu X))\Psi_{\mu j} + \frac{i}{4}e^\varphi\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu{}^i\gamma_5\gamma_\nu(L_i{}^j\partial_\rho X)\Psi_{\sigma j} \\
&+\frac{i}{4}e^\varphi\bar{\chi}_i\gamma^\mu(L_i{}^j\partial_\mu X)\chi^j + \frac{i}{4}e^\varphi\bar{\Omega}_i\gamma^\mu(L_i{}^j\partial_\mu X)\Omega^j - ie^\varphi\bar{\chi}_i\gamma^\mu\partial_\mu X\Omega^i
\end{aligned} \tag{50}$$

and the corresponding supertransformation laws are the following:

$$\begin{aligned}
\delta\Psi_{\mu i} &= 2D_\mu\eta_i + e^\varphi(L_i{}^j\partial_\mu X)\eta_j \\
\delta\chi^i &= -i\gamma^\mu[\partial_\mu\varphi\delta_i{}^j - e^\varphi(L_i{}^j\partial_\mu X)]\eta_j \\
\delta\Omega^i &= -i\gamma^\mu[e^\varphi(\partial_\mu X + \vec{L}(\vec{L}\partial_\mu X))\delta_i{}^j + D_\mu L_i{}^j]\eta_j \\
\delta\varphi &= (\bar{\chi}^i\eta_i) \quad \delta X = e^{-\varphi}[(\bar{\Omega}^i\eta_i) + (\bar{\chi}^i L_i{}^j\eta_j)] \quad \delta\vec{L} = (\bar{\Omega}^i(\vec{\tau})_i{}^j\eta_j).
\end{aligned} \tag{51}$$

If one adds an interaction with the vector multiplets, described in the previous subsection, the following additional terms in the Lagrangian arise:

$$\begin{aligned}
\Delta\mathcal{L} &= \frac{i}{4}e^\varphi\{(\bar{\Theta}_i\gamma^\mu L_i{}^j\partial_\mu X\Theta_j) + (\bar{\rho}_i\gamma^\mu L_i{}^j\partial_\mu X\rho_j)\} - \\
&- \frac{i}{4\sqrt{2}}e^{\sqrt{2}\hat{\varphi}}\{(\bar{\chi}^i\gamma^\mu\gamma_5\chi^i) + (\bar{\Omega}^i\gamma^\mu\gamma_5\Omega^i)\}\partial_\mu\hat{\pi} - \\
&- \frac{1}{8}e^{\hat{\varphi}/\sqrt{2}}\{(\bar{\Omega}^i\bar{\mathcal{Z}}(\sigma A)\varepsilon_{ij}\Omega^j) + \bar{\chi}^i\bar{\mathcal{Z}}(\sigma A)\varepsilon_{ij}\chi^j)\}.
\end{aligned} \tag{52}$$

In this, the whole Lagrangian (44, 45, 50, 52) is invariant under the supertransformations (47, 51).

3.3 Spontaneous symmetry breaking

The problem, we are interested in, is: if it is possible to break simultaneously supersymmetry and gauge symmetry in the way, described in the previous Section? Let us first consider the possibility of the supersymmetry breaking. For this one has to detach the hidden sector of the model and investigate its global symmetries. Let us divide the index M of the vector multiplets as $M = \{\tilde{M}, A\}$, $\tilde{M} = 1,2,3,4$, $g^{\tilde{M}\tilde{N}} = (-,-,+,+)$ and the index $\hat{A}$ of the hypermultiplets as $\hat{A} = \{\tilde{A}, A\}$, $\tilde{A} = 1,...,6$, $g^{\tilde{A}\tilde{B}} = (-,-,-,+,+,+)$. The hidden sector contains the following fields from the vector multiplets: ρ_i, $\hat{\varphi}$, $\hat{\pi}$ and $\{A_\mu^{\tilde{M}}, \Theta_i^{\tilde{M}}, \mathcal{Z}^{\tilde{M}}\}$ and the following fields from the hypermultiplets: φ, χ^i and $\{X^{\tilde{A}}, \vec{L}^{\tilde{A}}, \Omega^{i\tilde{A}}\}$. The scalar fields from the hypermultiplets, entering the hidden sector, parameterize the quaternionic manifold $SO(4,4)/SO(4)\otimes SO(4)$, in this the fields $X^{\tilde{A}}$ enter the Lagrangian through the divergency only. In [12, 14] it has been shown that the gauging of a part of this global translations leads to the spontaneous supersymmetry breaking with two arbitrary mass scales and vanishing cosmological constant.

The observable sector of the model contains the vector multiplets $(A_\mu, \Theta_i, \mathcal{Z})^A$ and the hypermultiplets $(X, \vec{L}, \Omega^i)^A$. Let us assume, just like it have been made in the previous section, that the fields from these multiplets lie in the Kac-Moody algebra (3) and divide index A: $A \to \{A, m\}$ with m being an infinite index. Then one can switch on the gauge interaction in the observable sector with all the fields from both types of the multiplets transforming under the same representation of algebra (3). For example, transformation laws for the fields $\mathcal{Z}$ have the form:

$$\delta\mathcal{Z}_m^A = f^{ABC}\mathcal{Z}_n^B\varepsilon_{m-n}^C \tag{53}$$

and the same for all other fields, besides the fields X_m^A, which have unhomogeneous term in the transformation laws:

$$\delta X_m^A = f^{ABC} x_n^B \varepsilon_{m-n}^C + i\mu m \varepsilon_m^A \tag{54}$$

The following substitutions into the Lagrangian of the model make it gauge-invariant:

$$\partial_\mu \mathcal{Z}_m^A \to \partial_\mu \mathcal{Z}_m^A + f^{ABC} A_{\mu n}^{\ B} \mathcal{Z}_{m-n}^C \tag{55}$$

and similar ones for all the fields except X_m^A and

$$\partial_\mu X_m^A \to \partial_\mu X_m^A + f^{ABC} A_{\mu n}^{\ B} X_{m-n}^C - i\mu m A_{\mu m}^{\ A}. \tag{56}$$

As usual in supergravities, switching on the gauge interaction spoils the invariance under the supertransformations and in order to restore it one has to add the following terms to the Lagrangian and to the supertransformation laws:

$$\begin{aligned}
\mathcal{L}'^F \ = \ & e^{-\hat{\varphi}/\sqrt{2}} \Big\{ -\frac{1}{4}\bar{\Psi}_{\mu i}\sigma^{\mu\nu}\varepsilon^{ij}\Delta_j{}^k\Psi_{\nu k} + \frac{i}{2\sqrt{2}}\bar{\Psi}_{\mu i}\gamma^\mu\bar{\Delta}_i{}^j\rho_j - \frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu\Delta_i^{\tilde{M}j}\Theta_j^{\tilde{M}} - \\
& -\frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu(\Delta_m^A)_i{}^j\Theta_{j-m}^{\ A} - \frac{i}{2}e^{\varphi}\bar{\Psi}_{\mu i}\gamma^\mu\{\bar{\Delta}_i^{\tilde{M}j}\mathcal{Z}^{\tilde{M}} + (\bar{\Delta}_{1m}^{\ A})_i{}^j\mathcal{Z}_{-m}^A\}\varepsilon_{jk}\chi^k - \\
& -\frac{i}{2}e^{\varphi}\bar{\Psi}_{\mu i}\gamma^\mu\varepsilon_{ij}\bar{\Delta}^{\tilde{A}}\Omega^{j\tilde{A}} + \frac{1}{\sqrt{2}}\bar{\rho}_j\varepsilon^{jk}\Delta_k^{\tilde{M}i}\Theta_k^{\tilde{M}} - \\
& -\frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu\varepsilon_{ij}\{e^{\varphi}f^{ABC}\bar{\mathcal{Z}}_n^B X_{m-n}^C\delta_k{}^j + f^{ABC}\bar{\mathcal{Z}}_n^B(L_{m-n}^C)_k{}^j + ie^{\varphi}\mu m\bar{\mathcal{Z}}_m^A\}\Omega^{jA}_{\ \ -m} + \\
& +\frac{1}{\sqrt{2}}\bar{\rho}_j\varepsilon^{jk}(\Delta_m^A)_k{}^i\Theta_{i-m}^{\ A} + \frac{1}{\sqrt{2}}\bar{\rho}_i\{\bar{\Delta}_j^{\tilde{M}i}\mathcal{Z}^{\tilde{M}} + (\bar{\Delta}_{1m}^{\ A})_i{}^j\mathcal{Z}_{-m}^A\}\chi^j - \frac{2}{\sqrt{2}}\bar{\rho}_i\bar{\Delta}^{\tilde{A}}\Omega^{i\tilde{A}} \\
& -\frac{1}{\sqrt{2}}\bar{\rho}_i\{e^{\varphi}f^{ABC}\bar{\mathcal{Z}}_n^B X_{m-n}^C\delta_j{}^i + f^{ABC}\bar{\mathcal{Z}}_n^B(L_{m-n}^C)_j{}^i - ie^{\varphi}\mu m\bar{\mathcal{Z}}_m^A\delta_j{}^i\}\Omega_{-m}^{jA} - \\
& -\frac{1}{4}\bar{\Theta}_i^M\varepsilon^i\Delta_j{}^k\Theta_k^M - \bar{\Theta}_i^{\tilde{M}}\Delta_j^{\tilde{M}i}\chi^j - e^{\varphi}\bar{\Theta}_{-m}^A\{(\Delta_{1m}^{\ A})_j{}^i + (\Delta_{3m}^{\ A})_j{}^i\}\chi^j + \\
& +f^{ABC}\bar{\Theta}_{i-m}^{\ A}\{e^{\varphi}X_{m+n}^B\delta_j{}^i + (L_{m+n}^B)_j{}^i\}\Omega_{-n}^{jC} - ie^{\varphi}\mu m\bar{\Theta}_{im}^A\Omega_{-m}^{iA} - \frac{1}{4}\bar{\chi}^i\bar{\bar{\Delta}}_i{}^j\varepsilon_{jk}\chi^k - \\
& -\bar{\chi}^i\varepsilon_{ij}\bar{\Delta}^{\tilde{A}}\Omega^{j\tilde{A}} - e^{\varphi}\bar{\chi}^i\varepsilon_{ij}\{f^{ABC}\bar{\mathcal{Z}}_n^B X_{m-n}^C - i\mu m\bar{\mathcal{Z}}_m^A\}\Omega_{-m}^{jA} + e^{\varphi}\bar{\Theta}_i^{\tilde{M}}K^{\tilde{M}\tilde{A}}\Omega^{i\tilde{A}} - \\
& -\frac{1}{4}\bar{\Omega}^{i\hat{A}}\bar{\bar{\Delta}}_i{}^j\varepsilon_{jk}\Omega^{k\hat{A}} + f^{ABC}\{\frac{1}{2}\bar{\Omega}^{iA}_{\ \ m}\varepsilon_{ij}\bar{\mathcal{Z}}_{-m-n}^B\Omega^{jC}_{\ \ n} + \frac{i}{4}\bar{\Psi}_{\mu i}\gamma^\mu\mathcal{Z}_n^B\bar{\mathcal{Z}}_{-m-n}^C\Theta_{im}^{\ A} - \\
& -\frac{1}{2}\bar{\Theta}_{im}^{\ A}\varepsilon^{ij}\mathcal{Z}_{-m-n}^B\Theta_{jn}^{\ C} + \frac{1}{2\sqrt{2}}\bar{\Theta}_{im}^{\ A}\mathcal{Z}_{-m-n}^B\bar{\mathcal{Z}}_n^C\varepsilon^{ij}\rho_j\}\Big\}
\end{aligned} \tag{57}$$

$$\begin{aligned}
\mathcal{L}'^B \ = \ & -\frac{1}{2}e^{-\sqrt{2}\hat{\varphi}}\Big\{\vec{\Delta}^{\tilde{M}}\vec{\Delta}^{\tilde{M}} + \vec{\Delta}_m^A\vec{\Delta}_{-m}^A + 2e^{2\varphi}|\vec{\Delta}^{\tilde{M}}\mathcal{Z}^{\tilde{M}} + \vec{\Delta}_{1m}^{\ A}\mathcal{Z}_{-m}^A|^2 + \\
& +|\Delta_{2\ m}^{abA}\mathcal{Z}_{-m}^A|^2 + e^{2\varphi}|\mathcal{Z}^{\tilde{M}}K^{\tilde{M}\tilde{A}}|^2 + e^{2\varphi}|f^{ABC}\mathcal{Z}_n^B X_{m-n}^C - i\mu m\mathcal{Z}_m^A|^2 + \\
& +|f^{ABC}\mathcal{Z}_n^B\vec{L}_{m-n}^C|^2 + \frac{1}{4}|f^{ABC}\mathcal{Z}_n^B\bar{\mathcal{Z}}_{m-n}^C|^2\Big\}
\end{aligned} \tag{58}$$

$$\begin{aligned}
\delta'\Psi_{\mu i} &= e^{-\hat{\varphi}/\sqrt{2}}\frac{i}{2}\gamma_\mu\varepsilon^{ij}\Delta_j{}^k\eta_k \qquad \delta'\chi^i = -e^{\varphi}e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}\{\Delta^{\tilde{M}j}{}_k\mathcal{Z}^{\tilde{M}} + (\Delta^A_m)_i{}^j\mathcal{Z}^A_{-m}\}\eta_k \\
\delta'\rho_i &= -e^{-\hat{\varphi}/\sqrt{2}}\frac{1}{\sqrt{2}}\Delta_i{}^j\eta_j \qquad \delta'\Theta_i^{\tilde{M}} = e^{-\hat{\varphi}/\sqrt{2}}\{\Delta^{\tilde{M}j}{}_i + \frac{1}{2}\mathcal{Z}^{\tilde{M}}\bar{\Delta}_i{}^j + \frac{1}{2}\bar{\mathcal{Z}}^{\tilde{M}}\Delta_i{}^j\}\eta_j \\
\delta'\Omega^{i\tilde{A}} &= \varepsilon^{ij}e^{-\hat{\varphi}/\sqrt{2}}\{\mathcal{Z}^{\tilde{M}}K^{\tilde{M}\tilde{A}}\delta_i{}^j + L^{a\tilde{A}}[\Delta^{a\tilde{M}}\mathcal{Z}^{\tilde{M}}\delta_i{}^j + \Delta_{1n}^{aA}\mathcal{Z}^A_{-m}\delta_i{}^j - \Delta_2^{ab}{}^A_m\mathcal{Z}^A_{-m}\tau^b{}_i{}^j]\}\eta_j \\
\delta'\Theta_{im}^A &= e^{-\hat{\varphi}/\sqrt{2}}\{(\Delta^A_m)^j_i + \frac{1}{2}\mathcal{Z}^A_m\bar{\Delta}_i{}^j + \frac{1}{2}\bar{\mathcal{Z}}^A_m\Delta_i{}^j\}\eta_j + \frac{1}{2}e^{-\hat{\varphi}/\sqrt{2}}f^{ABC}\mathcal{Z}^B_n\bar{\mathcal{Z}}^C_{m-n}\eta_i \\
\delta'\Omega^{iA}{}_m &= \varepsilon^{ij}e^{-\hat{\varphi}/\sqrt{2}}\{e^{\varphi}[f^{ABC}\mathcal{Z}^B_nX^C_{m-n} - i\mu m\mathcal{Z}^A_m + \vec{L}^A_m(\vec{\Delta}^{\tilde{M}}\mathcal{Z}^{\tilde{M}} + \vec{\Delta}_{1n}{}^B\mathcal{Z}^B_{-n})]\delta_j{}^k + \\
+ & \quad [f^{ABC}\mathcal{Z}^B_n(L^C_{m-n})_i{}^j - L^{aA}{}_m\Delta_2^{ab}\tau^b{}_j{}^k]\}\eta_k, \qquad (59)
\end{aligned}$$

where the following notations are used:

$$\begin{aligned}
\Delta^a &= e^{\varphi}\Delta^{a\tilde{M}}\mathcal{Z}^{\tilde{M}} + e^{\varphi}(\Delta_1^a)^A_m\mathcal{Z}^A_{-m} - \frac{1}{2}\varepsilon^{abc}(\Delta_2^{bc})^A_m\mathcal{Z}^A_{-m} \\
\tilde{\Delta}^a &= e^{\varphi}\Delta^{a\tilde{M}}\mathcal{Z}^{\tilde{M}} + e^{\varphi}(\Delta_1^a)^A_m\mathcal{Z}^A_{-m} + \frac{1}{2}\varepsilon^{abc}(\Delta_2^{bc})^A_m\mathcal{Z}^A_{-m} \\
\vec{\Delta}^{\tilde{M}} &= K^{\tilde{M}\tilde{A}}\vec{L}^{\tilde{A}} \qquad (\vec{\Delta}_1)^A_m = f^{ABC}X^B_n\vec{L}^C_{m-n} + i\mu m\vec{L}^A_m \\
(\Delta_2^{ab})^A_m &= f^{ABC}L^{aB}_{\ n}L^{bC}_{\ m-n}. \qquad (60)
\end{aligned}$$

It is seen that the scalar field potential of the model has the minimum corresponding to the vanishing vacuum expectation values of the scalar fields from the observable sector, in this its value is the following:

$$V_0 = \frac{1}{2} < \{(K^{\tilde{M}\tilde{A}}\vec{L}^{\tilde{A}})^2 + |\mathcal{Z}^{\tilde{M}}K^{\tilde{M}\tilde{A}}|^2 + 2|\mathcal{Z}^{\tilde{M}}K^{\tilde{M}\tilde{A}}\vec{L}^{\tilde{A}}|^2\} > . \qquad (61)$$

One can choose the following vacuum expectation values for the fields of the hidden sector, consistent with the constraints on the scalar fields, $< \mathcal{Z}^{\tilde{M}} >= (1, i, 0, 0)$ and $< L^{a\tilde{A}} >= \delta^{a\tilde{A}}$. Also, let us choose the parameters $K^{\tilde{M}\tilde{A}}$ of the local translations in the form: $K^{\tilde{M}\tilde{A}} = M_1\delta^{1\tilde{M}}\delta^{1\tilde{A}} + M_2\delta^{2\tilde{M}}\delta^{2\tilde{A}}$. Now it is easy to check that vacuum expectation value of the scalar potential equals zero, which corresponds to the vanishing cosmological constant. In this, the gravitini mass matrix takes the form:

$$M^{ik} = -\frac{1}{2}\varepsilon^{ij} < \Delta_j{}^k >= \frac{1}{2}\begin{pmatrix} M_1 + M_2 & 0 \\ 0 & M_1 - M_2 \end{pmatrix} \qquad (62)$$

and we have spontaneous supersymmetry breaking with two arbitrary mass scales and, in particular, with the possibility of the partial super-Higgs effect $N = 2 \rightarrow N = 1$.

From the bosonic Lagrangian one can obtain, taking into account constraints (42) and (48) on the fields $\mathcal{Z}^M$ and $\vec{L}^{\hat{A}}$, the following mass terms for the scalar fields of the model:

$$\begin{aligned}
\mathcal{L}^s_M &= -\frac{1}{2}\, M_1^2L^{1A}{}_mL^{1A}{}_{-m} + M_2^2L^{2A}{}_mL^{2A}{}_{-m} + M_1^2\mathcal{X}^A_m\mathcal{X}^A_{-m} + M_2^2\mathcal{Y}^A_m\mathcal{Y}^A_{-m} + \Big\} \\
&\quad + (\mu m)^2[\vec{L}^A_m\vec{L}^A_{-m} + \mathcal{Z}^A_m\bar{\mathcal{Z}}^A_{-m}] + 4i\mu m[M_1L^{1A}{}_m\mathcal{X}^A_{-m} + M_2L^{2A}{}_m\mathcal{Y}^A_{-m}]\Big\}. \qquad (63)
\end{aligned}$$

Corresponding mass terms for the fermionic and vector fields of the observable sector are the following:

$$\mathcal{L}^f_M = \frac{1}{2}\bar{\Theta}_{im}^AM^{ij}\Theta_{j-m}^A - i\mu m\bar{\Theta}_{im}^A\Omega^{iA}{}_{-m} + \frac{1}{2}\bar{\Omega}^{iA}{}_mM_{ij}\Omega^{jA}{}_{-m}, \qquad (64)$$

Table 1: The mass spectrum in the observable sector

	vector fields	spinor fields	scalar fields
$m > 0$	μm	$\frac{M_1+M_2}{2} + \mu m$ $\lvert\frac{M_1+M_2}{2} - \mu m\rvert$ $\lvert\frac{M_1-M_2}{2} + \mu m\rvert$ $\lvert\frac{M_1-M_2}{2} - \mu m\rvert$	$M_1 + \mu m$ $\lvert M_1 - \mu m\rvert$ $M_2 + \mu m$ $\lvert M_2 - \mu m\rvert$ μm
$m = 0$	0	$\frac{M_1+M_2}{2}$ $\lvert\frac{M_1-M_2}{2}\rvert$	M_1 M_2 0

where the mass matrices $M_{ij} = M^{ij}$ are the same as in (62) and $\mathcal{L}_M^v = \frac{1}{2}(\mu m)^2 A_{\mu\nu}{}_m^A A_{\mu\nu}{}_{-m}^A$.

After the diagonalization the mass spectrum of the model is the following (see Table). At the lowest level ($m = 0$) the vector fields are massless and for each one we have two massless scalars, two scalars with masses equal to M_1 and two ones with M_2 as well as two spinors with masses equal to $(M_1 + M_2)/2$ and the same number of spinors with $(M_1 - M_2)/2$. In the case of partial super-Higgs effect $N = 2 \to N = 1$ ($M_1 = M_2 = M$), all these fields form massless vector $N = 1$ supermultiplets and massive (with masses equal to M) chiral $N = 1$ supermultiplets, as it should be.

At the level with the level number m for each massive vector field with mass (μm) we have pairs of scalar fields with masses equal to $(M_1 - \mu m)$, $(M_1 + \mu m)$, $(M_2 - \mu m)$, $(M_2 + \mu m)$ and (μm) and the pairs of spinor fields with masses equal to $\frac{M_1+M_2}{2} + \mu$, $\frac{M_1+M_2}{2} - \mu$, $\frac{M_1-M_2}{2} + \mu$ and $\frac{M_1-M_2}{2} - \mu$. Again, in the case when $N = 2$ supersymmetry breaks to $N = 1$ ($M_1 = M_2 = M$), all these fields form massive vector $N = 1$ supermultiplets with masses equal to (μm) and the same number of massive scalar $N = 1$ multiplets with masses $(M + \mu m)$ and $(M - \mu m)$.

It is not difficult to obtain analogous results for the case of the generalized Kac-Moody algebra (22). In this case at the lowest level part of the vector fields acquire masses, while the part of the vector fields, associated with the generators of H subgroup, remains massless exactly in the same way as it would be when the gauge symmetry breaks $G \to H$.

There is only one mass scale in the model with such mechanism of the gauge symmetry breaking. Therefore, if one investigates a unified model with a gauge group, such as $SU(5)$, then the offered scheme can be used only to break the unification gauge group to the gauge group of the Standard Model, for example, $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$. Then one should again use the Higgs mechanism.

One useful observation can be made from the mass spectrum of the model under consideration. In the case, when, for example, $\mu \approx M_1$, there is a number of the "light" scalar fields in the model with masses equal to $(M_1 - \mu)$. In the models with a finite dimensional gauge

group all such fields acquire masses M_1 and M_2 [12, 14] which are close to the mass scale of the supersymmetry breaking $N = 2 \to N = 1$ and there is no way to lower its values, while in the model considered a kind of inverse Higgs mechanism is operative and these "light" particles can play a role in the low energy phenomenology, for example, as the Higgs fields in the breaking $SU(2) \otimes U(1) \to U(1)_{em}$.

Conclusion

So we have managed to construct a class of $N = 2$ supergravity models, allowing simultaneous spontaneous breaking of both the supersymmetry and the gauge symmetry. The supersymmetry was broken with two arbitrary mass scales and vanishing cosmological constant. For the gauge symmetry breaking specific properties of the gauge theories with the infinite dimensional Kac-Moody algebras were used and it was shown that such a mechanism worked in the case of $N = 2$ supergravity. It seems to be natural to use this scheme for the breaking of an unification gauge group, such as $SU(5)$. One of the interesting results, obtained as a byproduct of the whole construction, is that after the gauge symmetry breaking some scalar fields, which after the breaking of the supersymmetry acquire masses, close to the scale of $N = 2$ supersymmetry breaking, can be made "light" and can be used for the breaking of the electro-weak gauge group like Higgs fields. For this inverse Higgs mechanism to be operative, the mass scale of the $N = 2 \to N = 1$ supersymmetry breaking have to be close to the scale of unified gauge symmetry breaking exactly as $N = 1 \to N = 0$ supersymmetry breaking scale is expected to be close to electro-weak one. Note, at last, that the quaternionic non-linear σ-model we have chosen for the hypermultiplets allows one to consider the generalization of our present work to the case of quaternionic models, based on non-symmetric quaternionic spaces, constructed in [13, 14].

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